Section 1: Cover Sheet

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14. ABSTRACT

The objectives of this project were to explore decomposition algorithms that solve optimization models under uncertainty. In order to accommodate a variety of future scenarios, our algorithms are designed to address large scale models. The main accomplishments of the project can be summarized as follows.

- design and evaluate decomposition methods for stochastic mixed-integer programming (SMIP) problems (Yuan and Sen [2008])
- accelerate stochastic decomposition (SD) as a prelude to using SD for SMIP as well as a multi-stage version of SD (Sen et al [2007], Zhou and Sen [2008]).
- develop a theory for parametric analysis of mixed-integer programs, and provide economically justifiable estimates of shadow prices from mixed-integer linear programming models (Sen and Genc [2008])

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Section 2: Objectives

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- develop a theory for parametric analysis of mixed-integer programs, and provide economically justifiable estimates of shadow prices from mixed-integer linear programming models (Sen and Genc [2008])

The first two bullets relate to stochastic programming, whereas the last bullet addresses one of the long-standing open questions in discrete optimization, namely, parametric analysis in MILP models. This paper (listed as [1]) is likely to have a long term impact on a variety of fields including discrete optimization, operations research, and computational economics.

Section 3: Accomplishments / New Findings: Research Highlights and Relevance to the Air Force Mission

This section presents the principal research accomplishments of this project. It is organized into three main sub-sections, each representing a paper that has been submitted for publication. These new nuggets of knowledge cover a wide array of discrete optimization research, ranging from new theory, algorithms, and computational experiments. *Results that are of particular relevance to the Air Force are highlighted* [1].

3.1 Stochastic Mixed-Integer Programming Algorithms

Stochastic Mixed Integer Programs (SMIP) are recognized as one of the most formidable classes of mathematical programming problems. Not only are there significant challenges due to potentially large number of scenarios, but, SMIP with integers in the second stage give rise to a non-convex and discontinuous recourse function that may be difficult to optimize. Ahmed et al [2004] provide an illustration of how the presence of integer variables in the second stage leads to extremely complicated (non-convex and discontinous) recourse functions. Over the past few years, there have been significant

[1] In this section, when a reader encounters one or more sentences in *italics*, he/she should interpret the content as being directly relevant to the Air Force and its Technological Challenges.

advances in the design of algorithms for solving SMIP problems (Sen [2005] for a survey). However, computational implementations, and results for large scale problems have been slow in coming. Some exceptions to this comment are papers by Alonso-Ayuso et al [2003], and Ntaimo and Sen [2006]. In the former paper, the authors present a branch-and-fix algorithm which they use to solve a supply chain planning problem, whereas the latter presents a computational comparison using supply chain as well as large scale stochastic server location problems. Research funded through the current grant presents new computational approaches (Yuan and Sen [2008]) for decomposition-based branch-and-cut method (D^2-BAC, Sen and Sherali [2006]). The main computational tools developed by Yuan and Sen [2008] (submitted to *INFORMS Journal on Computing*) include a streamlined cut generation procedure which itself can be interpreted as a stochastic linear program to choose cut D^2 coefficients that are effective for a variety of second stage scenarios. In addition, our approach overcomes the need to solve certain linear programs that convexify the value function approximations. These enhancements yield some of the fastest solution times reported on server location problems, with speed-ups averaging a factor of about 2. *As a result of such speed-ups, a collaborative DARPA project between AT&T and Telecordia is planning to implement these tools for a new generation of design tools for communications networks*.

3.2 Accelerating Stochastic Decomposition

Since stochastic linear programming forms an integral part of SMIP as well as multi-stage decision-making under uncertainty, we have studied methods that accelerate the stochastic decomposition (SD) algorithm. Specifically, we study two issues: a) are there any conditions under which the regularized version of SD generates a unique solution? and b) is there a way to modify the SD algorithm so that a user can trade-off solution times with solution quality? The second issue addresses the scalability of SD for very large scale problems for which computational resources may be limited and the user may be willing to accept solutions that are “nearly optimal". These issues become critical in the solution of SMIP problems where a large number of stochastic linear programming approximations may be necessary. The same considerations arise in the solution of multi-stage stochastic linear programs in which the value function may be approximated by re-sampling previously observed scenarios (as in particle filtering). In Sen et al [2008] (submitted to *Computers and Operations Research*), we show that by using bootstrapping (re-sampling) the regularized SD algorithm can be accelerated without significant loss of optimality. Another paper which applies these ideas to multi-stage stochastic linear programming is currently under preparation for *Mathematical Programming* (Zhou and Sen [2008]).

3.3: Parametric Analysis for MILP

One of the more important open problems in trade-off analysis deals with pricing integral activities or discrete decisions (as well as continuous decisions). This is vital for cost-effective programs in which large fixed-costs play a key role. One may model these choices using mixed-integer linear programming (MILP), and pricing these activities rely on shadow prices for MILP.

It is well known that in general MILP problems present duality gaps and dual variables (as part of the price system) are not unique and not as conveniently interpreted. These issues have been visited for almost fifty years starting with Gomory and Baumol (1960) and subsequently by a number of other authors; however, these issues remained unresolved. Yet, shadow prices continue to play a prominent role in economic theory and practice. For instance, starting in 2008, economists for the British government have agreed to a certain shadow price schedule for pricing carbon emissions in evaluating new projects until the year 2050[2]. Without access to tools that provide justifiable estimates of shadow prices, the social value of such programs will remain questionable.

Research associated with this project (Sen and Genc [2008]) provides an important step in allocating charges of indivisible goods, and moreover, characterizes shadow prices for resources in 0-1 MILP problems. In particular we address the following question: Is it possible to prescribe shadow prices that are unique, and recover the total cost of inputs, even when the underlying model includes indivisibilities that are modeled using integer variables? Our approach is guided by two ideas:

a) We propose a new measure of two-sided shadow prices for binary MILP problems in which the optimal objective value function is both non-convex and discontinuous. This measure also generalizes the two-sided shadow prices used in the context of linear programming (LP) (Gal (1997)).
b) Balas (1979) (see also Sherali and Adams (1990)) shows that for 0-1 MILP problems, the convex hull of feasible points can be generated by using certain linear programs in which the contributions of each technology can be traced.

For the case of 0-1 MILP problems, we show that these two ideas lead to the existence of unique shadow prices for 0-1 MILP problems. As in O'Neill et al (2005), we explain the optimal cost of a primal MILP as the value of outputs plus the total "start-up" price of technologies. However, in contrast to the above paper, the prices suggested by our research are guaranteed to be non-negative. We also study the value function of MILP problems with a view towards obtaining accurate shadow prices. In order to do so, we define a new class of two-sided shadow prices. We develop an LP-based methodology for calculating them. While there is a modest computational cost due to the solution of LPs, the shadow price estimates can be expected to be stronger in general. These prices also have the shadow price interpretations similar to those in classic linear programming. In the process of developing this framework, we also provide an interpretation of implied constraints in the form of productivity requirements that must be satisfied for integer programming problems. The paper reporting these results (Sen and Genc [2008]) will be submitted to the Computational Economics special issue of *Operations Research*.

Section 4: Personnel Supported

Suvrajeet Sen (PI) and a graduate student (Yang Yuan) were supported through this grant.

[2] See http://www.defra.gov.uk/environment/climatechange/research/carboncost/pdf/HowtouseSPC.pdf

Section 5: Publications

(all papers listed below can be obtained by sending e-mail to the PI at sen.22@osu.edu)

S. Sen and T. Genc, Non-negativity of Start-up Prices and Uniqueness of Shadow Prices in a Resource Allocation Model with Indivisibilities, prepared for submission to computational economics special issue of *Operations Research*.

S. Sen, Z. Zhou and K. Huang, Enhancements of Two-Stage Stochastic Decomposition, submitted to *Computers and Operations Research*, (abridged version to appear in *Stochastic Programming: The State of the Art,* Volume edited in honor of George B. Dantzig, G. Infanger editor).

Y. Yuan and S. Sen, Enhanced cut generation methods for decomposition-based branch-and-cut algorithms for two-stage stochastic mixed-integer programs, submitted to *INFORMS Journal on Computing*.

Z. Zhou and S. Sen, Multi-stage Stochastic Decomposition, prepared for submission to *Mathematical Programming*.

Citations to the Literature

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